

Weak Equivalence Principle and Propagation of the Wave Function in Quantum Mechanics

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The propagation of the wave function of a particle is characterised by a group and a phase velocity. The group velocity is associated with the particle's classical velocity, which is always smaller than the speed of light, and the phase velocity is associated with the propagation speed of the wave function phase and is treated as being unphysical, since its value is always greater than the speed of light. Here we show, using Sciama's Machian formulation of rest mass energy, that this physical interpretation, for the group and the phase velocity of the wave function, is only valid if the weak equivalence principle strictly holds for the propagating particle, except for the photon. In case this constraint is released the phase velocity of the wave function could acquire a physical meaning in quantum condensates.

INTRODUCTION — A quantum particle is a dual entity with corpuscular and wave properties. On the one side classically, a corpuscle is essentially characterized by its position \vec{r} , inertial mass m_i , electric charge q , linear momentum, $\vec{p} = m_i \vec{v}$, and kinetic energy $E_c = p^2/2m$. On the other side, a classical wave has the following main attributes: Wavelength λ , propagation speed w , frequency $\nu = w/\lambda$, Amplitude A , Intensity $I = A^2$, Energy E , linear momentum \vec{p} . A corpuscle is a localized entity, in contrast with a wave which has a certain extension through space. Quantum mechanics accounts for the dual aspect of quantum entities by postulating a level of reality more fundamental than the particle itself. This consists in a complex wave function Ψ , which is *not directly observable*. The value of Ψ associated with a moving particle, at a point with coordinates, x , y , and z at time t , is related with the probability $d\pi$ to observe the particle at this particular point.

$$d\pi = |\Psi|^2 dx dy dz = \Psi \Psi^* d\tau \quad (1)$$

Where $d\tau = dx dy dz$ is the infinitesimal volume element in the neighborhood of the point (x, y, z) , and Ψ^* is the complex conjugate of Ψ . The wavelength of the wave function is the so called de Broglie wavelength, which is inversely proportional to the particle's linear momentum $p = m_{0i}v/(1 - v^2/c^2)^{1/2}$:

$$\lambda = \frac{h}{m_{0i}v} \left(1 - \frac{v^2}{c^2}\right)^{1/2} \quad (2)$$

where h is Planck's constant, and m_{0i} is the particles proper inertial mass. Assuming that the entire energy of the particle, $E = E_0/(1 - v^2/c^2)^{1/2}$, can be converted into one single quanta of electromagnetic energy, the frequency of the wave function ν is related with the particle's rest mass energy through the well know Planck's formula:

$$\nu = \frac{E_0}{h \left(1 - \frac{v^2}{c^2}\right)^{1/2}} \quad (3)$$

where E_0 is the particle's rest mass energy.

MACHIAN INTERPRETATION OF THE REST MASS ENERGY — Starting from Mach's principle, which asserts that there is a holistic-type connection between the local laws of physics and the large scale properties of the universe, Sciama in [1] introduced the relation

$$c^2 = \frac{2GM}{R} \quad (4)$$

where R and M are the radius and the mass of the universe. Einstein's relationship linking proper energy E_0 and proper mass m_0 then takes the form

$$E_0 = m_0 c^2 = \frac{2GMm}{R} \quad (5)$$

which can be interpreted as a statement that the proper inertial energy that is present in any physical particle is due to the gravitational potential energy of all the matter in the universe acting on the particle. Therefore the mass m_0 appearing in eq.(5) should be regarded as the proper gravitational mass m_{0g} of the particle.

$$E_0 = m_{0g} c^2 \quad (6)$$

GROUP AND PHASE VELOCITY OF THE WAVE FUNCTION — To compute the propagation speed w of the wave function, one starts from:

$$\lambda = \frac{w}{\nu} \quad (7)$$

Substituting Eq. (2) and Eq.(3) in Eq.(7) we obtain,

$$\frac{h}{m_{0i}v} = w \frac{h}{E_0} \quad (8)$$

Substituting Eq.(6) in Eq.(8) one gets

$$\frac{m_{0g}}{m_{0i}} = \frac{wv}{c^2} \quad (9)$$

For a realistic normalizable wave function forming a wavepacket, corresponding to the propagation of a free particle, it is easy to demonstrate [2][3][4] that v and w

correspond to the group and the wave velocity of the wavepacket respectively. The former coincides with the velocity of the particle, the later corresponds to the velocity of propagation of the phase of the wave function.

WEAK EQUIVALENCE PRINCIPLE AND WAVE'S FUNCTION PHASE VELOCITY — The Weak Equivalence Principle is one of the main foundations of the theory of general relativity. It means the constancy of the ratio between the inertial and the gravitational mass m_i and m_g respectively of a given physical system.

$$\frac{m_g}{m_i} = \iota \quad (10)$$

where ι is a constant. This implies, in classical physics, that the possible motions in a gravitational field are the same for different test particles.

Since the weak equivalence principle cannot be demonstrated on a purely theoretical basis, it can only be justified by experiment. Thus ι can only be obtained from experiments. Current experimental tests of the weak equivalence principle [6] [7], indicate that the gravitational and inertial masses of any classical physical system should be equal to each other

$$m_g/m_i = \iota = 1. \quad (11)$$

This is observed within a relative accuracy of the Eötvös-factor, $\eta(A, B)$ less than 5×10^{-13} .

$$\eta(A, B) = (m_g/m_i)_A - (m_g/m_i)_B < 5 \times 10^{-13} \quad (12)$$

where A and B designate two different bodies, or the same body at different times.

DISCUSSION — Substituting Eq.(11) in Eq.(9) we deduce the well known relation between the group and phase velocity of the wave function and the speed of light in vacuum [5].

$$wv = c^2 \quad (13)$$

Since according to the laws of special relativity the group velocity, i.e. the particle's classical velocity, cannot exceed the speed of light c , the phase velocity is necessarily higher than the speed of light. This is generally interpreted as demonstrating the un-physical nature of the phase velocity of the wave function. *Thus neither the wave function or its wave velocity can be directly detected and none of them can be associated with the propagation of information at supra-luminal speeds.* It is nevertheless worth stressing that this physical interpretation of the wave function phase velocity is only possible if the weak equivalence principle holds for the propagating particle, i.e., if the inertial and gravitational mass of the particle are exactly equal to each other.

For the sake of completeness, let us investigate what would be the consequences of requiring that the wave function phase velocity is physical, i.e. that $w = c$. Substituting this condition in Eq.(9), we conclude that

$$\frac{m_{0g}}{m_{0i}} = \frac{v}{c} \quad (14)$$

Assuming that the weak equivalence principle holds, i.e. $m_{0g}/m_{0i} = 1$, we conclude that Eq.(14) leads to $v = c$, hence it only applies to photons and not to other material particles. If instead we assume a possible violation of the weak equivalence principle, in order to allow the phase velocity to be physical for all types of particles, then Eq.(14) leads us to understand this symmetry breaking as being equivalent to a rotation in Minkowsky spacetime between two inertial observers in relative motion to each other.

In the case of superconductors and superfluids the particles making the condensate have canonical momentum \vec{p} proportional to the gradient of the phase of the wave function φ .

$$\hbar \nabla \varphi = \vec{p} \quad (15)$$

Assuming an experimental context in which the canonical momentum $\vec{p} = m\vec{v}$, one deduce from Eq.(15), that in superconductors and superfluids, the group and the phase velocity of the wave function, \vec{v} and \vec{w} respectively, are equal to each other.

$$\vec{v} = \frac{\hbar \nabla \varphi}{m} = \vec{w} \quad (16)$$

Substituting Eq.(16) in Eq.(9) one obtains:

$$\frac{m_{0g}}{m_{0i}} = \left(\frac{v}{c}\right)^2 \quad (17)$$

Since in general $v \ll c$ in the Earth laboratory, this would mean that the weak equivalence principle is strongly broken for Cooper pairs in superconductors and for Helium atoms forming the superfluid condensate in superfluid Helium. This conclusion is streamlined with other published work indicating a possible breaking of the weak equivalence principle for Cooper pairs in superconductors [8] and for superfluid vortices in rotating superfluid Helium [9], resulting from a breaking of gauge invariance in these physical systems. Substituting Eq.(17) in Eq.(12) we obtain the Eötvös factor characterizing the breaking of the weak equivalence principle in quantum condensates, when compared with the normal state of these materials for which $m_{0g}/m_{0i} = 1$.

$$\eta = 1 - \left(\frac{v}{c}\right)^2 \quad (18)$$

CONCLUSION—In this short note we have demonstrated that assuming the Machian interpretation of proper energy, which leads to interpreting the inertial energy content of a particle as being due to its gravitational interaction with the entire universe, we can assign the unphysical nature of the phase velocity of the particle's wave function to the fact that it complies with the weak equivalence principle, Eq.(9).

Requiring that the wavefunction phase velocity is physical by propagating at the speed of light, leads to assume either that this is only possible for the case of the photon, or that the weak equivalence principle is violated by

quantum particles. Since there is no experimental evidence that this is the case, as measured in Collela Overhauser Werner (COW)[10] experiments with cold neutron interferometry, we conclude that the phase velocity of the wave function is unphysical except for the photon.

For the special case of superconductors and superfluids, we showed that the phase velocity of the conden-

sate's wave function would be physical since being equal to the condensate's group velocity, Eq.(16). This leads to consider a possible breaking of the weak equivalence principle in these materials. This conclusion is also supported by other research work on the consequences of the breaking of gauge invariance in superconductors and superfluids[8] [9].

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